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DIFFUSION FROM A LINE SOURCE IN A TURBULENT BOUNDARY LAYER

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Abstract—Diffusion from a continuous line source is calculated by modifying a method originally proposed on the basis of Lagrangian similarity considerations to describe the mean position of an ensemble of particle releases. Good agreement is found with measurements of the characteristic size and maximum concentration throughout the intermediate-stage of diffusion in a developing turbulent boundary layer.

a_1,	constant in (15);
a_2,	constant in (17);
b,	Batchelor's constant defined in (3);
B,	constant in (26);
B_1,	constant in (27);
ΔB,	shift of the logarithmic profile in (30);
c,	concentration;
C_{max},	maximum concentration near the boundary;
$f(\xi)$,	defined in (11);
p,	defined in (32);
Q,	discharge of the source per unit width;
q_z,	flux in the z-direction;
u,	mean longitudinal velocity;
V^*,	shear velocity;
x,	distance from the source;
$\bar{x}$,	mean longitudinal position of an ensemble of single particle releases;
z,	distance normal to boundary;
z_0,	characteristic length defined in (16);
$\bar{z}$,	mean vertical position of an ensemble of single particle releases;
B,	mean vertical position of particles from a continuous source at a cross section defined in (11);
δ,	momentum boundary-layer thickness;
λ,	characteristic size of the diffusing boundary layer, $c(\lambda) = C_{max}/2$;
ε,	eddy diffusivity;
ξ,	dimensionless parameter, $\xi = z/\lambda$.

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1. INTRODUCTION

PREDICTION of diffusion in turbulent shear flows is usually based on approximate semi-empirical analogies, similarities, and phenomenological laws. The earliest theory of turbulent diffusion of Taylor [1] assumed, in analogy to molecular diffusion, that the flux of the diffusing matter q_z is proportional to an eddy diffusivity ε and the local concentration gradient

$$q_z = -\varepsilon \frac{\partial c}{\partial z}. \tag{1}$$

When employing such a model it is hoped that ε is a local function of the velocity field so that its value at a point can be specified regardless of the position of the source or the size of the diffusion cloud. Many investigators have assumed, for example, that the eddy diffusivity for shear flows is equal to the eddy viscosity, defined by $\bar{\varepsilon} = -\overline{u'v'}/(\partial u/\partial z)$. Later studies have shown, however, that this is not always the case ([2], Chapter 5). The study of diffusion in homogeneous turbulence has indicated, for example, that ε, as defined by (1), is independent of the position of the source only if the distance to the source is large compared to the Lagrangian integral scale of the turbulent motion. Estimates of the Lagrangian integral scale in boundary layers [3] suggest that it is larger than ten boundary-layer thicknesses. Thus, it is not fully justified to employ such a model in predicting diffusion behind a source in a boundary layer. Indeed, evaluation of ε from experimental data [4] shows that it is not only a function of the distance normal to the boundary but depends as well on the size of the diffusion plume relative to the boundary-layer thickness, or equivalently, the distance from the source.

In 1957, Batchelor [5] introduced Lagrangian similarity considerations to predict the turbulent motion of particles in self-preserving shear flow. The Lagrangian similarity hypothesis was applied to that region of the boundary layer where the mean-velocity $u(z)$ can be expressed as a function of the shear velocity, V^* and a measure of the turbulent scale, z_0, in the form

$$u = V^* g(z/z_0). \tag{2}$$

In this region the hypothesis can be summarized (Cermak [6]) as follows: "For a marked particle which is at $z = h$ when $t = 0$, the statistical properties of particle motion at time t depend only upon V^* and $t - t_v$, where t is of the order of h/V^* or larger, and t_v is a virtual time origin".

As a consequence of this hypothesis two equations describing the change of the mean vertical position, $\bar{z}$, and the mean longitudinal position, $\bar{x}$, for an ensemble of single-particle releases may be written [5]:

$$\mathrm{d}\bar{z}/\mathrm{d}t = bV^*, \tag{3}$$

where b is a universal constant, termed Batchelor's constant, and

$$\mathrm{d}\bar{x}/\mathrm{d}t = V^* g(\bar{z}/z_0) = u(\bar{z}). \tag{4}$$

By eliminating the time variable $\mathrm{d}t$ between (3) and (4) the following equation describing the trajectory of the mean position is obtained:

$$\mathrm{d}\bar{z}/\mathrm{d}\bar{x} = bV^*/u(\bar{z}). \tag{5}$$

Equation (5) can be integrated when $g(z/z_0)$ is known. For the case of a logarithmic velocity profile

$$u/V^* = \log(z/z_0)/k \tag{6}$$

one obtains

$$\mathrm{d}\bar{x}/\mathrm{d}\bar{z} = \log(\bar{z}/z_0)/bk \tag{7}$$

and

$$bk\bar{x}/z_0 = (\bar{z}/z_0)\log(\bar{z}/z_0) - \bar{z}/z_0 + \text{constant}. \tag{8}$$

The constant of integration in (8) is determined by the method of introduction of the tracer into the flow.

In 1963, Cermak [6] applied the Lagrangian similarity hypothesis to estimate the variation of the ground level concentration downstream from continuous point and line sources. Such an estimate is obtained by relating the concentration due to a continuous source to the probability density function of single release particles at the same point. If it is assumed that the probability density function for single-particle releases, which is expected to be a universal function of $(x - \bar{x})/\bar{z}$ and $(z - \bar{z})/\bar{z}$, has a sharp maximum at $x = \bar{x}$ it is found that the ground level concentration $C_{\max}$ at a distance x downstream from a continuous line source is

$$C_{\max} \propto Q/[V^* \bar{z} g(\bar{z}/z_0)], \tag{9}$$

or

$$C_{\max} \propto Q/[\bar{z}u(\bar{z})] \tag{10}$$

where Q is the discharge of the source (per unit width).

Most of the experimental data reported in the literature describe the variation of $C_{\max}$ in the form $C_{\max} \propto x^m$ where x is the distance downstream from the source. Equations (8) and (9), and the corresponding equations for point sources, enable one to calculate m when b and the constant of integration are known. Cermak has found reasonably good agreement between the theoretical prediction of m and data from a large number of wind tunnel and field experiments of mass and heat diffusion, using $b = 0{\cdot}1$. The value of b, however, depends to a large extent on the estimate of the constant in (8). Pasquill [7] suggested that b is approximately 0·4, in agreement with a suggestion by Ellison [8] that $b = k$, where k is von Kármán's constant in (6).

The mean position $\bar{z}(\bar{x})$ cannot be measured easily. Measurements have usually been made of the concentration distribution at different stations downstream from continuous sources and thus the mean position of the particles $\bar{Z}$ [defined in equation (14)] at each station x can be easily calculated. The purpose of this work is to analyze the data on diffusion from a line source at ground level presented by Poreh and Cermak [4] (referred to hereinafter as the data), and to examine whether simple equations like (5) can also be employed to calculate $\bar{Z}$, and thus to predict the rate of diffusion, in developing boundary layers.

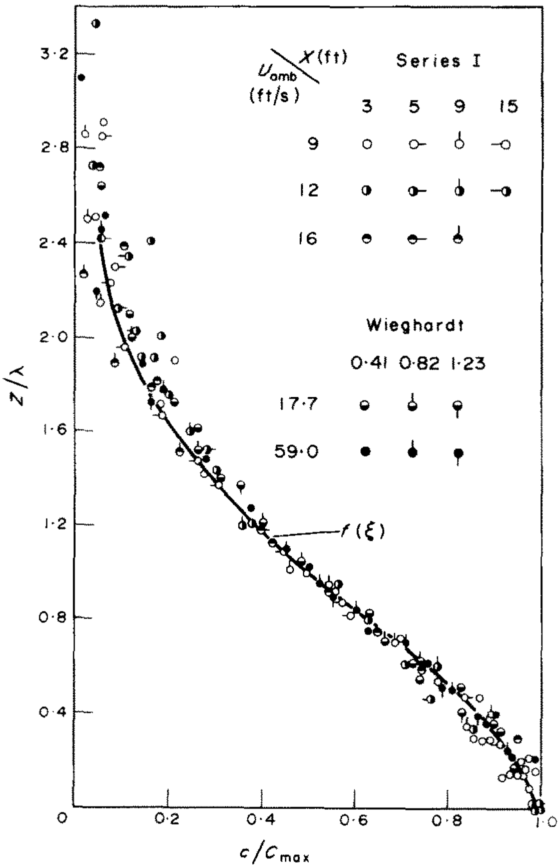

FIG. 1. Dimensionless concentration profiles in the intermediate zone [4].

2. THE DIFFUSION PATTERN IN TURBULENT BOUNDARY LAYERS

The experimental data on diffusion from a line source at the wall in a neutrally buoyant, turbulent boundary layer indicate four stages of diffusion [4]:

(1) The initial stage close to the source.

(2) The intermediate stage, which extends 20–40 boundary-layer thicknesses downstream, in which the concentration profiles are found to be approximately similar (see Fig. 1) in the sense that

$$c/C_{max} = f(\xi) \tag{11}$$

where

$$\xi = z/\lambda \quad \text{and} \quad f(1) = 0{\cdot}5.$$

The variation of λ with the distance x from the source in this stage was found experimentally to be described by

$$\lambda = 0{\cdot}076 \,.\, x^{0{\cdot}8} \quad (x \text{ and } \lambda \text{ in cm}) \tag{12}$$

and the measurements of the maximum concentration were approximated by

$$C_{max}U = 26{\cdot}2Qx^{-0.9} \tag{13}$$

where U is the ambient velocity. [Note that (12) and (13) are empirical equations with dimensional constants.]

(3) A transition stage with somewhat slower growth of the diffusion boundary layer.

(4) A final stage in which the diffusion is limited by the growth of the developing boundary layer and the size of the plume is proportional to the thickness of the boundary layer.

The present discussion will be limited to the intermediate zone.

In view of the similarity, the mean position of particles at any cross section $\bar{Z}$, which is defined by

$$\bar{Z} = \int_0^\infty cz\,\mathrm{d}z \Big/ \int_0^\infty c\,\mathrm{d}z \tag{14}$$

is given by

$$\bar{Z} = a_1 \lambda \tag{15}$$

where

$$a_1 = \int_0^\infty \xi f(\xi)\,\mathrm{d}\xi \Big/ \int_0^\infty f(\xi)\,\mathrm{d}\xi \cong 0{\cdot}76. \tag{16}$$

The velocity measurements in the boundary layer presented in [3] were found to be described by the power law

$$u/U = (z/\delta)^{1/n}; \qquad n = 7. \tag{17}$$

Substituting (11) and (17) into the continuity equation

$$\int_0^\infty uc\,\mathrm{d}z = Q,$$

the following equation results:

$$Q = UC_{\max}\lambda(\lambda/\delta)^{1/n} \int_0^\infty \xi^{1/n} f(\xi)\,\mathrm{d}\xi. \tag{18}$$

The value of the definite integral in (18) is estimated from Fig. 1 to be around 1·05. Since $\bar{Z} = 0{\cdot}76\lambda$ and $u(\bar{Z}) = U\,.\,(\bar{Z}/\delta)^{1/n}$, it follows that

$$C_{\max} = Q/[a_2 \bar{Z}\, u(\bar{Z})], \tag{19}$$

where a_2 is approximately 1·45. Comparing equations (10) and (19) one finds that

$$\bar{z}\, u(\bar{z}) \propto \bar{Z}\, u(\bar{Z}). \tag{20}$$

which is satisfied, or course, if

$$\bar{Z} = \bar{z}. \tag{21}$$

As indicated earlier, Ellison suggested that the coefficient b equals k. His estimate is based on the use of the eddy diffusivity model (1). From this model one may obtain the following approximate relationship:

$$\frac{\mathrm{d}\bar{Z}}{\mathrm{d}t} = -\int_0^\infty \varepsilon \frac{\partial c}{\partial z}\,\mathrm{d}z \Big/ \int_0^\infty c\,\mathrm{d}z. \tag{22}$$

Integration of (22) by parts gives

$$\frac{\mathrm{d}\bar{Z}}{\mathrm{d}t} = \int_0^\infty \frac{\partial \varepsilon}{\partial z} c\,\mathrm{d}z \Big/ \int_0^\infty c\,\mathrm{d}z. \tag{23}$$

If u is described by (6), $u'v'$ is assumed a constant and ε is assumed to be equal to the eddy viscosity, then $\partial\varepsilon/\partial z$ is a constant and (23) reduces to

$$\mathrm{d}\bar{Z}/\mathrm{d}t = kV^*, \tag{24}$$

where k is the von Kármán's constant.

Equation (23) describes the mean upward velocity of the particles at a given section and we have therefore denoted it by $\mathrm{d}\bar{Z}/\mathrm{d}t$ rather than by $\mathrm{d}\bar{z}/\mathrm{d}t$. Thus, Ellison's suggestion that b in (3) equals k implies that the mean vertical change of $\bar{Z}$ equals that of $\bar{z}$, which is consistent with (21). Equation (21) is also supported by Pasquill [6], who compared a few field observations of $\bar{Z}$ with values of $\bar{z}$ calculated from (8) with $b = k$ and found them to be virtually identical.

3. CALCULATION OF THE DIFFUSION RATE IN THE INTERMEDIATE ZONE

In view of the previous discussion it is suggested that the diffusion rate behind a continuous line source can be calculated with reasonable accuracy by an equation, similar to (5),

$$\mathrm{d}\bar{Z}/\mathrm{d}x = bV^*/u(\bar{Z}). \tag{25}$$

and that the maximum concentration can then be calculated by (19). Several limitations should, however, be recognized when applying (25) to boundary layers:

(i) The logarithmic profile does not extend beyond $z/\delta = 0{\cdot}15$.

(ii) The boundary layer thickness increases in the downstream direction.

(iii) All turbulent quantities, such as $\overline{u'v'}/V^{*2}$ and $\overline{v'^2}/V^{*2}$, begin to decrease beyond

$z/\delta = 0{\cdot}15$. It is therefore reasonable to expect that b in (25) would also decrease as $\bar{Z}/\delta$ becomes large.

These limitations, which indicate that the flow field does not possess a self-preserving character, raise doubts whether one could, a priori, expect to find a similarity of the concentration profiles. The experimental data suggests, however, that the dimensionless distribuyion of the diffusing matter is not affected to a large extent by the mild changes in the velocity field. On the basis of this observation one is justified to assume that the inhomogeneity of the field could be accounted for, with sufficient accuracy, by using at each station, the local values of the parameters which appear in (25).

Landweber [9] has calculated the development of a turbulent boundary layer and the shear distribution along a flat plate with zero pressure gradient. His calculations are based on the distinction of three regions. A viscous sublayer near the wall, which does not affect the diffusion except very close to the source and may be totally neglected at high Reynolds numbers; a logarithmic region, $zV^*/\nu > 30$ and $z/\delta < 0{\cdot}15$ in which

$$u/V^* = \log(V^*z/\nu)/k + B \qquad (26)$$

or

$$u/V^* = U/V^* - B_1 + \log(z/\delta)/k; \qquad (27)$$

and an outer zone where

$$(U - u)/V^* = F(z/\delta). \qquad (28)$$

It was pointed out later (see Discussion in [9]) that

$$F(z/\delta) = K(1 - z/\delta)^2, \qquad K \cong 9{\cdot}8. \qquad (29)$$

Landweber's calculations are in very good agreement with the experimental data. Assuming that the velocity field is not affected by the diffusing matter, we shall use his solution to calculate V^* and δ downstream from the source. Equations (26)–(29) will then be used to determine $u(\bar{Z})$.

As indicated earlier, it is expected that the coefficient b in (25) should decrease when $\bar{Z}/\delta$ becomes large. The same conclusion follows from (23) when the measurements of ε across the boundary layer are used to estimate $d\bar{Z}/dt$. Measurements of ε by Lin [10], shown in Fig. 2, can be approximated by

$$\varepsilon/\delta V^* = 0{\cdot}09\{1 - \exp[-k(z/\delta + 2z^2/\delta^2)/0{\cdot}09]\} \qquad (30)$$

with $k = 0{\cdot}384$. The limit of (30) for small z is kV^*z in agreement with the conventional assumption for this region which was used by Ellison.

Substituting ε from (30) in (23) gives

$$d\bar{Z}/dt = kV^*p(\lambda/\delta) \qquad (31)$$

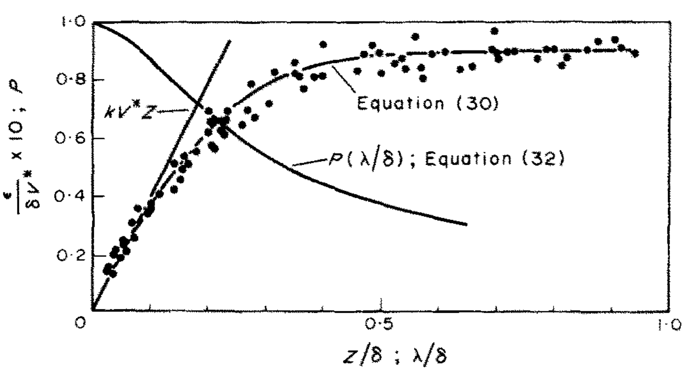

FIG. 2. Distribution of edgy viscosity according to Lin [10].

where

$$p(\lambda/\delta) = \int_0^\infty (1 + 4\xi\lambda/\delta) \times \exp\left[-k(\xi\lambda/\delta + 2\xi^2\lambda^2/\delta^2)/0{\cdot}09\right] \times f(\xi)\,d\xi \Big/ \int_0^\infty f(\xi)\,d\xi. \quad (32)$$

The dependence of p on λ/δ given by (32) has been calculated numerically and is shown in Fig. 2. Because of the limitations of (1), (32) cannot be considered to be a reliable quantitative estimate of the value of b in (25). Indeed, when used in the integration of (25) the diffusion rate did not match the experimental data. It does, however, support the intuitive argument that b is not a constant and should decrease when the diffusion plume emerges from the logarithmic layer. As a rough estimate of b it is proposed that use be made of the linear relation

$$b = k(1 - \bar{Z}/\delta) \quad (33)$$

which matches Ellison's suggestion near the wall and yields an average between the upward velocity near the wall and the upward velocity near the edge of the boundary layer, which is probably close to zero, when $\bar{Z}/\delta = 0{\cdot}5$.

Equation (25) was integrated numerically using a predicator-corrector method. The value of $\bar{Z}(x_{i+1})$ was first estimated and then the change of $\bar{Z}$ was calculated using the average of the slope $\Delta\bar{Z}/\Delta x$ at x_i and the estimated slope at x_{i+1}. The value of Δx in the numerical integration was taken to be smaller than $\delta/2$ near the source and δ at larger distances. Further reduction of Δx did not affect the results significantly.

The results of the numerical calculations are shown in Figs. 3–6.

4. ANALYSIS OF THE RESULTS

The change of the characteristic height of the diffusing plume λ with the distance x from the source was calculated by integrating (25), assuming an initial value of $\lambda_0 = 0{\cdot}00485$ m ($\bar{Z} = 0{\cdot}00365$ m) at $x = 0{\cdot}1$ m in agreement with (12). Figure 3 describes the calculated change of λ with x for different initial values. The calculations confirm the expectation that when $(x - x_0)/\lambda_0$ becomes large, the value of λ should be independent of the initial value λ_0.

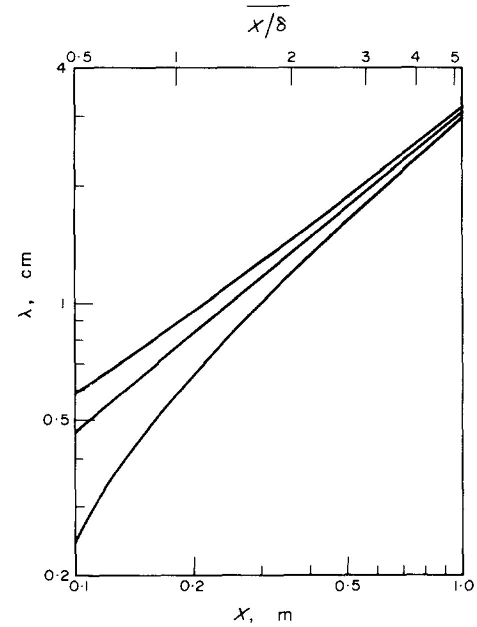

FIG. 3. Variation of λ with x for different initial conditions ($U = 3{\cdot}7$ m/s).

The values calculated of λ with $b = k$, $b = 0{\cdot}8k$, and $b = (1 - \bar{Z}/\delta)k$ for $U = 3{\cdot}7$ m/s are compared with (12) in Fig. 4. The curve describing the results with $b = k$ follows the experimental power law only when λ/δ (or $\bar{Z}/\delta$) is small. It deviates, however, from the experimental line farther downstream, confirming the expectation that the mean upward velocity is smaller in that region. Integration of (25) with a smaller but constant value of b does not seem to be satisfactory either, as is suggested by the curve for $b = 0{\cdot}8k$. On the other hand, the deviation of the computed values with $b = (1 - \bar{Z}/\delta)k$ from (12) is very small throughout the intermediate zone.

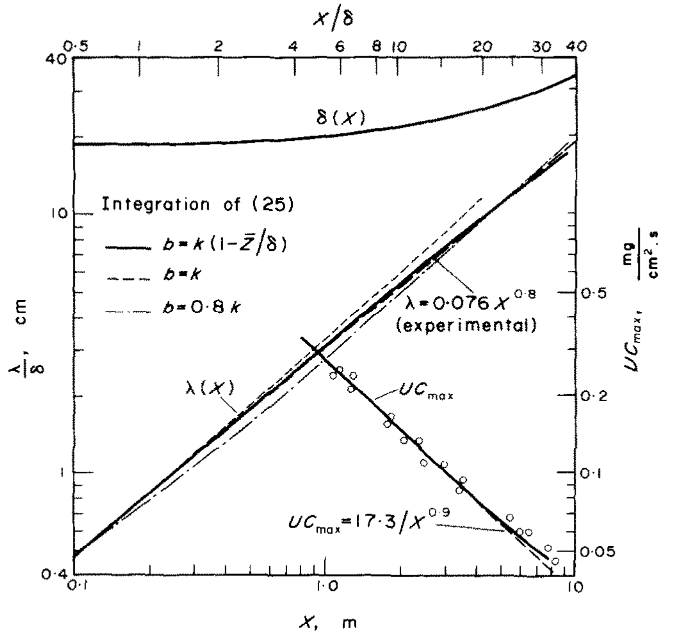

FIG. 4. Development of the diffusion boundary layer in the intermediate zone. (Data and power laws from [4].)

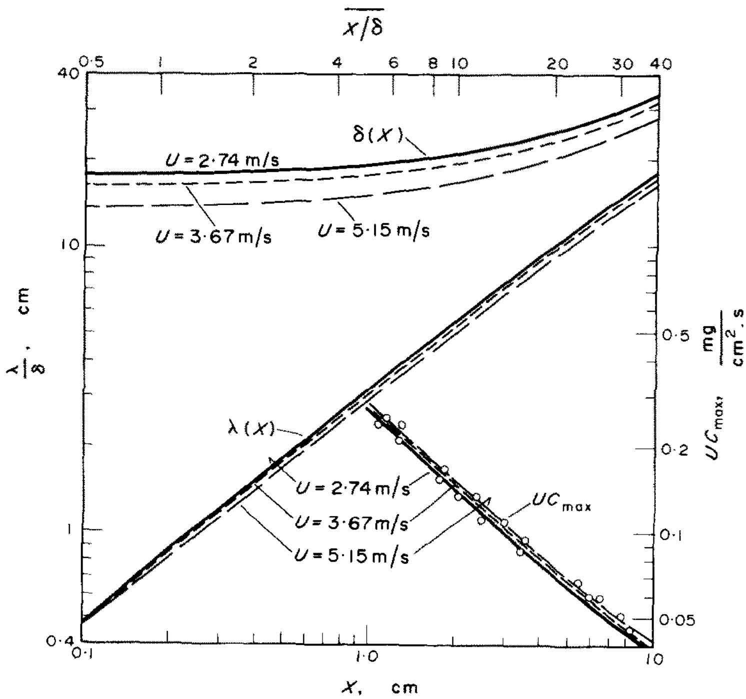

FIG. 5. Development of the diffusion boundary layer at different ambient velocities (data from [4]).

Figure 4 also shows the measured and the computed values of UC_{max} with $b = (1 - \bar{Z}/\delta)k$. The computed curve, shown by a solid line, coincides with (13) in the intermediate zone. When λ/δ becomes closer to its asymptotic value, 0·64, the computed curve is closer to the measurements than the power law.

Most of the data presented in [4] were measured at three different ambient velocities: $U = 2{\cdot}74$, 3·67 and 5·15 m/s. The corresponding values of δ at the position of the source were estimated to be 0·178, 0·163 and 0·137 m. The experimental data did not show consistent differences between the values of λ and UC_{max} in those three runs. Figure 5 shows the calculated values of λ and UC_{max}, with $b = k(1 - \bar{Z}/\delta)$, for these three velocities. The corresponding values of $\delta(x)$ are also shown. One sees that the difference between the computed values for the three cases is also very small. Such small differences could have not been distinguished experimentally.

It should be pointed out that there is no general agreement as to the value of the constants in (26) and (27). Equation (22) was therefore integrated using two consistent sets of constants: the set $k = 0{\cdot}385$, $B = 4{\cdot}0$, $B_1 = 2{\cdot}0$, which was used by Landweber [9], and the set $k = 0{\cdot}4$, $B = 5{\cdot}5$, $B_1 = 2{\cdot}35$. The same initial values of λ_0 and δ_0 at $x_0 = 0{\cdot}1$ m were used in both cases. The differences between λ and UC_{max} throughout the intermediate zone in the two cases were found to be smaller than two per cent, although the shear velocity V^* calculated with the different sets of constants differed by approximately 5 per cent.

The calculated values of $\overline{\lambda/\delta}$ are plotted in Fig. 6 vs. x/δ. The parameter $\overline{x/\delta}$ is defined by

$$\overline{x/\delta}(x) = \int_0^x \delta(x)^{-1}\,\mathrm{d}x. \qquad (34)$$

The calculations were carried up to $\lambda/\delta = 0{\cdot}64$, although it is clear that the proposed model is valid only in the intermediate zone. One sees that the relative size of the diffusing plume is almost independent of the ambient velocity. The theoretical curves are in good agreement with the experimental data throughout the intermediate zone (only data from Series I of [4] are shown), and describe the slower growth of the plume in the transition region. On the other hand, the broken line in Fig. 6, which describes the values of λ/δ computed with $b = k$, gives too large a diffusion rate beyond $\lambda/\delta = 0{\cdot}2$.

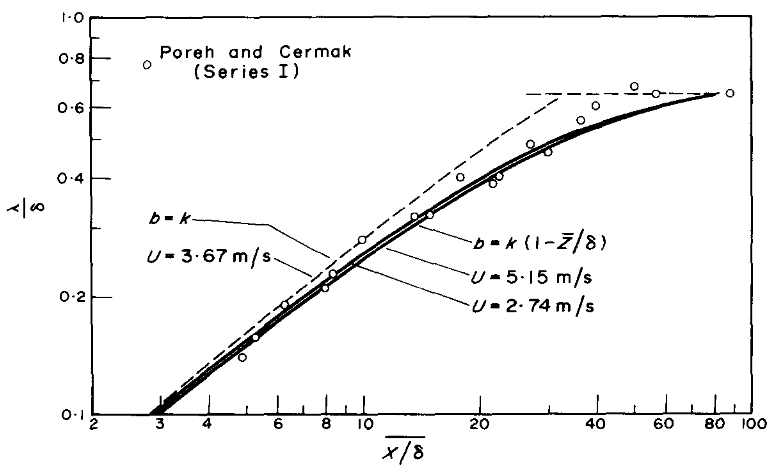

FIG. 6. Relative growth of the diffusion boundary layer.

5. EFFECT OF ROUGHNESS OR POLYMER ADDITIVES ON THE DIFFUSION RATE

Surface roughness leaves the structure of the turbulent flow away from the wall unchanged but decreases the thickness of the viscous sub-layer ([2], Chapter 7). Equation (26) in the case of rough surfaces becomes

$$u/V^* = \log(V^*z/\nu)/k + B + \Delta B \qquad (35)$$

where ΔB is a negative number depending on the Reynolds number of the roughness. Drag reducing polymers do not change the structure of the logarithmic and the outer regions either. The log law remains valid except that a positive value of ΔB is observed, indicating an increase of the thickness of the viscous sub-layer [11]. In both cases (27) and (28) remain unchanged. The ratio of U/V^* increases, however, in the case of polymers, and decreases in the case of rough surfaces.

Since the structure of the logarithmic region and the outer region is unchanged, (25), (33) and (19) can be used to predict the diffusion rate and the maximum concentrations in such flows when ΔB is known. The effect of the roughness or polymer additives is apparent from the form of (25). As $d\bar{Z}/dx$ is approximately proportional to V^*/U, diffusion increases in the case of rough surfaces and decreases in cases of drag reduction.

Figure 7 shows the growth of the diffusion boundary layer as calculated by (25) and (33) for a smooth boundary ($\Delta B = 0$), a rough boundary ($\Delta B = -4$), and for flow of drag reducing polymers ($\Delta B = 4$).

The same value of $\lambda = 0{\cdot}22$ mm at $x = 2$ mm, $U = 5$ m/s and $\nu = 10^{-6}$ m/s were used in the three cases. The initial value of δ at that point was also assumed to be the same; $\delta = 4{\cdot}14$ mm, giving $U/V^* = 20{\cdot}1$, $23{\cdot}75$ and $27{\cdot}3$ for $\Delta B = -4$, 0 and 4.

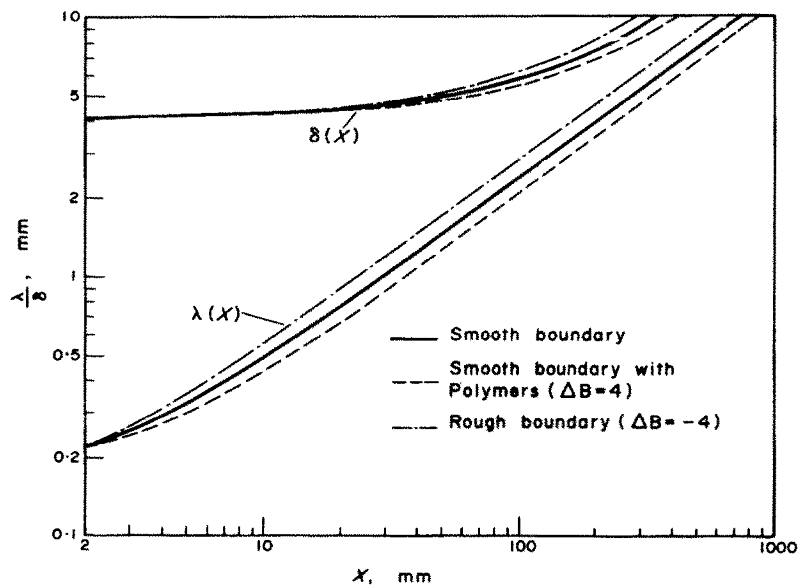

FIG. 7. Effect of surface roughness and drag reducing additives.

One sees from Fig. 7 that at large distances from the source the values of λ at a given distance x, scale proportionally to V^*/U.

7. SUMMARY AND CONCLUSIONS

It is suggested, on the basis of the observed similarity of the concentration profiles downstream of a continuous line source at ground level, that the mean vertical height of particles $\bar{Z}$ at a distance x from the source is approximately equal to the mean vertical height of an ensemble of single particle releases having the same mean distance from the source. Accordingly, the rate of growth of the diffusing boundary layer has been calculated by integrating the equation,

$$d\bar{Z}/dx = bV^*/u(\bar{Z}), \tag{25}$$

originally proposed on the basis of the Lagrangian similarity hypothesis to describe the trajectory of the mean position of an ensemble of particles. Comparison with experimental data shows that b is a constant, approximately equal to the von Kármán's constant k as suggested by Ellison [8], only when the diffusion boundary layer is completely submerged in the logarithmic layer of the velocity field. When the size of the diffusing boundary layer becomes larger, the diffusion rate decreases as a result of the reduced turbulent mixing in the outer region. Integration of (25) with $b = k(1 - \bar{Z}/\delta)$ gives a reasonable agreement with the experimental data throughout the intermediate zone. Numerical integration of (25) appears to be a simple method for predicting the diffusion rate in developing boundary layers.

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DIFFUSION À PARTIR D'UNE SOURCE LINÉAIRE DANS UNE COUCHE LIMITE TURBULENTE

Résumé—La diffusion à partir d'une source linéaire est calculée en modifiant une méthode déjà proposée et basée sur des considérations de similitude lagrangienne pour décrire la position moyenne d'un ensemble de particules. Un bon accord est trouvé avec les mesures de la taille caractéristique et de la concentration maximale dans toute la région intermédiaire de diffusion dans une couche limite turbulente en développement.

DIFFUSION VON EINER LINIENQUELLE IN EINE TURBULENTE GRENZSCHICHT

Zusammenfassung—Es wird die Diffusion von einer kontinuierlichen Linienquelle berechnet, indem eine ursprünglich auf der Basis der Lagrangeschen Ähnlichkeit beruhende Methode modifiziert wird, so dass sich die mittlere Lage eines Kollektivs von freigesetzten Teilchen bestimmen lässt.

Es zeigte sich, dass gute Übereinstimmung mit Messungen der charakteristischen Grösse und der maximalen Konzentration im ganzen Übergangsbereich der Diffusion in einer sich entwickelnden turbulenten Grenzschicht herrscht.

ДИФФУЗИИ ОТ ЛИНЕЙНОГО ИСТОЧНИКА В ТУРБУЛЕНТНОМ ПОГРАНИЧНОМ СЛОЕ

Аннотация—Диффузия от непрерывного линейного источника рассчитывается с помощъю модифицированного метода, основанного на соображениях подобия Лагранжа и первоначально предложенного для описания среднего положения ансамблей дискретных частиц. Установлено хорошее соответствие с измерениями характерного размера и максимальной концентрации в промежуточной стадии диффузии в развивающемся турбулентном пограничном слое.